Optical conductivity of a clean metal near Ising-nematic quantum critical point

Songci Li

School of Physics, Department of Physics, Tianjin University

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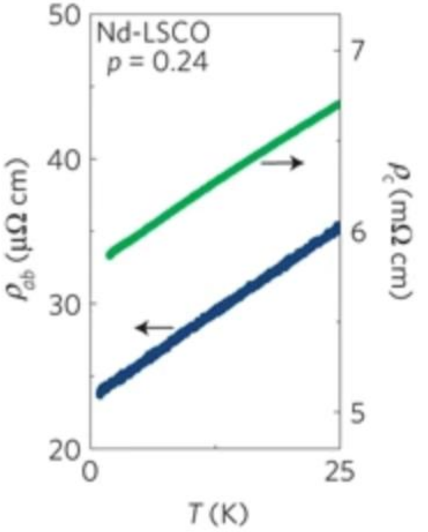
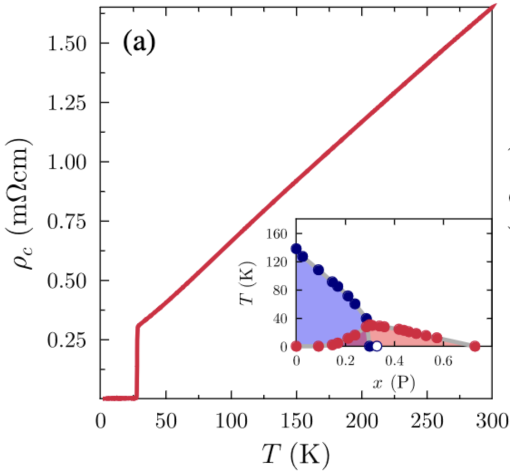
- Landau Fermi liquid (FL) theory provides a paradigm for understanding electron transport in metallic systems

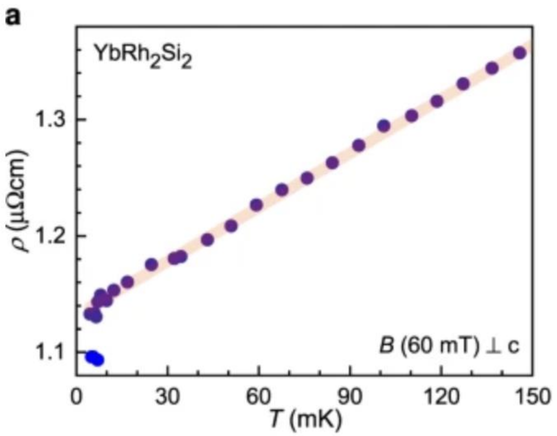
- However, nowadays many correlated electron systems show deviations from FL behavior. For example, *T*-linear resistivity observed in different electron systems



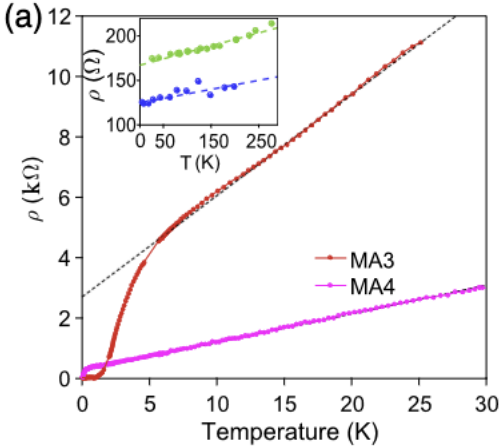
$La_{1.6-x}Nd_{0.4}Sr_xCuO_4$
Nature Physics 5,31 (2009)



$BaFe_2(As_{1-x}P_x)_2$
PRL 121, 197002 (2018)



$YbRh_2Si_2$
Nat Commun 12, 4341 (2021)



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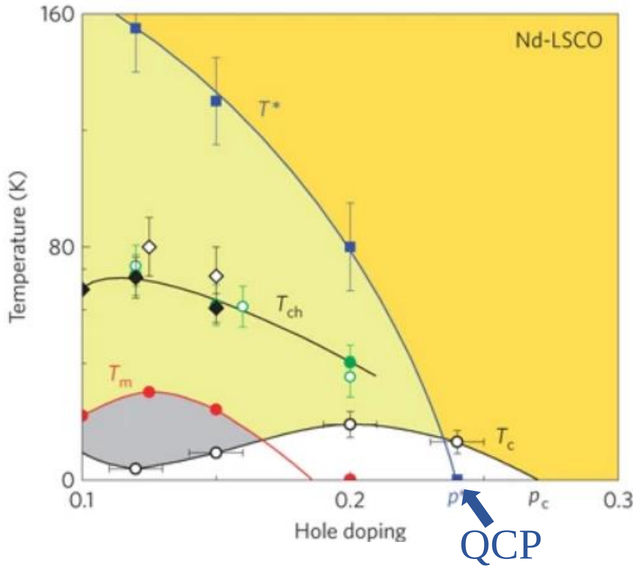
PRL 124, 076801 (2020)



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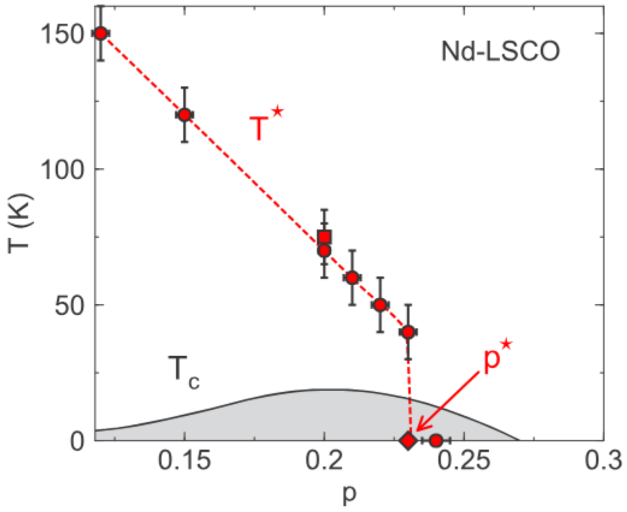
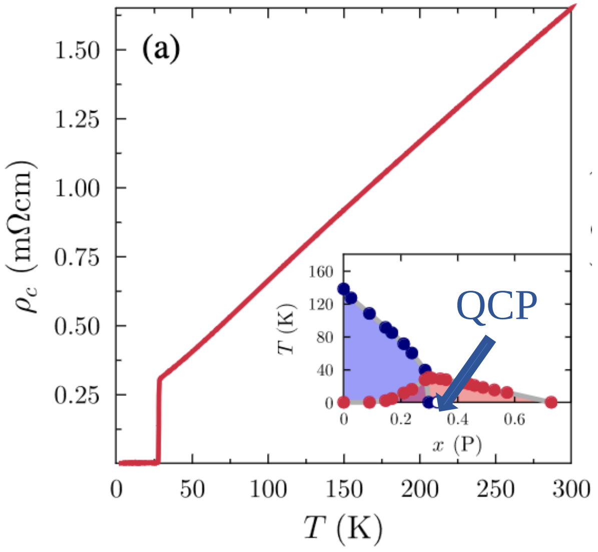
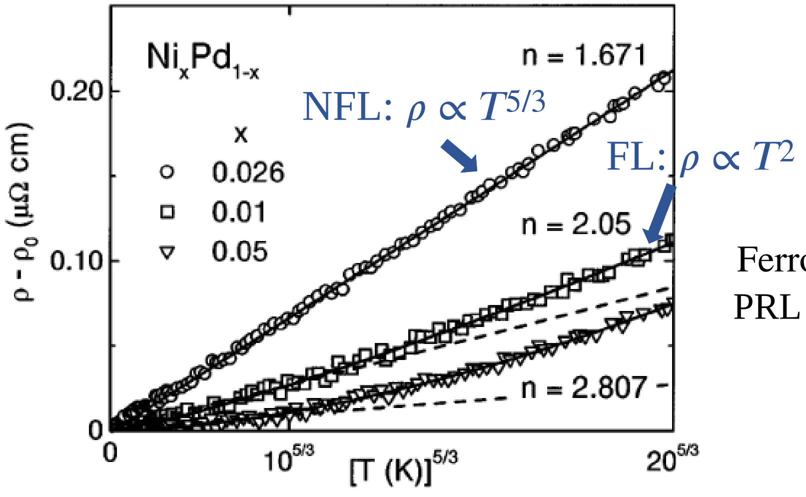


$La_{1.6-x}Nd_{0.4}Sr_xCuO_4$
PRB 95,224517 (2017)

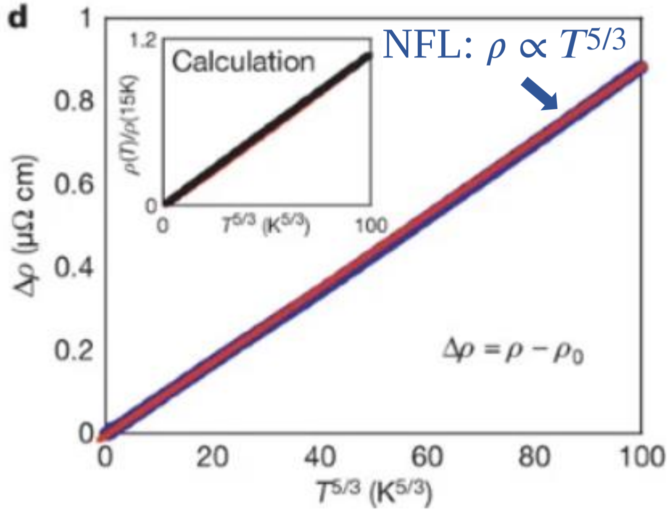


$BaFe_2(As_{1-x}P_x)_2$
PRL 121, 197002 (2018)

Other examples: itinerant ferromagnets



Ni_xPd_{1-x}
Ferromagnetic QCP
PRL 82,4268 (1999)



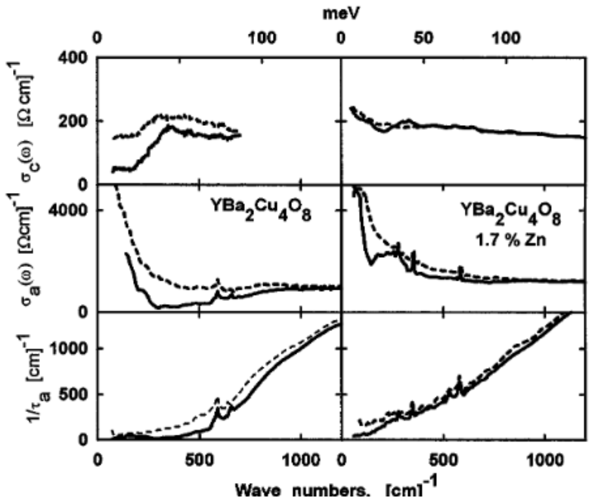
$ZrZn_2$
Ferromagnetic QCP
Nature 455,1220 (2008)

- Understanding the transport properties of electrons near QCP remains one of the challenges in the study of strongly correlated electron systems

- In addition to dc resistivity, optical responses of correlated electron systems also provide valuable information on the dynamics of electrons near QCP through optical conductivity $\sigma(\Omega, T)$

  Explore wide frequency range and extract possible Ω/T scaling in $\sigma'(\Omega, T)$

e.g. $\sigma'(\Omega) \propto 1/\Omega^{\alpha}$ with ($\alpha \approx 1$) observed for underdoped/optimally doped cuprates and iron-based superconductors, probably dual to T-linear dc resistivity (although other exponents of α are observed), see
N. Armitage, arXiv 0908.1126;
D. Basov et al. RMP 83, 471 (2011);
D. Basov, T. Timusk, RMP 77,721 (2005) ;
D. van der Marel et al. Ann. Phys. 321 (2006) 1716-1729;
D. Maslov, A. Chubukov, Rep. Prog. Phys. 80(2017) 026503…



YBCO
Basov et.al. PRL 77, 4090 (1996)

Outline

- Part I: Introduction

- Part II: Optical conductivity of a metal near Ising-nematic quantum critical point

1. *Songci Li*, Prachi Sharma, Alex Levchenko, Dmitrii Maslov, PRB 108, 235125 (2023)

2. Yasha Gindikin, *Songci Li*, Alex Levchenko, Alex Kamenev, Andrey Chubukov, Dmitrii Maslov, arXiv: 2406.10503, accepted in PRB

- Part III: Summary

Part I: Introduction

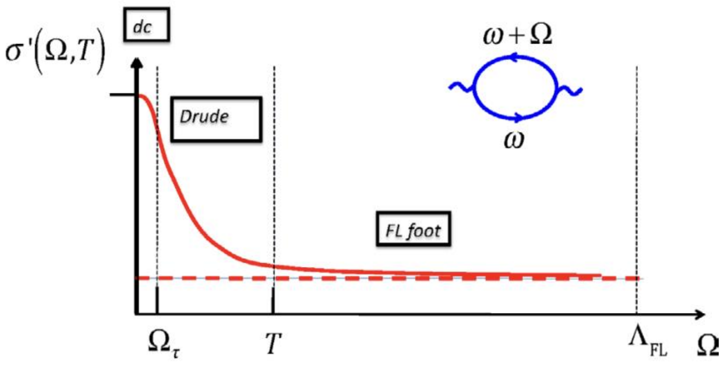
- Optical conductivity $\sigma(\Omega, T)$ is commonly used probe to extract electron dynamics in correlated materials

- Drude formula (phenomenological): $\sigma'(\Omega, T) \propto \frac{1}{\Omega^2 \tau_J(\Omega,T)}$ at $\Omega \gg 1/\tau_J$

  FL: $\tau_J^{-1}(\Omega, T) \propto max\{\Omega^2, T^2\}$ $\Longrightarrow$ $\sigma'(\Omega \gg T) \propto$ const; $\sigma'(\Omega \ll T) \propto T^2/\Omega^2$

- Optical conductivity of a FL: $\sigma'(\Omega, T) \propto \frac{\Omega^2 + 4\pi^2 T^2}{\Omega^2}$

  (Gurzhi, Sov. Phys. JETP 35, 673(1959); Maslov, Chubukov, Rep. Prog. Phys. 80(2017) 026503; PRB 86, 155137 (2012))



Maslov, Chubukov, Rep. Prog. Phys. 80(2017) 026503

- Optical conductivity of a NFL, e.g. a clean metal near Ising-nematic QCP ($D=2, z=3, q=0$):

$$1/\tau_q \backsim \Sigma'' \propto \Omega^{2/3} \Longrightarrow \frac{1}{\tau_J} \backsim \frac{q^2}{k_F^2}\Sigma'' \propto \Omega^{2/3}\Omega^{2/3} = \Omega^{4/3}$$

then based on Drude formula, $\sigma'(\Omega,0) \propto \frac{1}{\Omega^2 \tau_J} \propto \Omega^{-2/3}$

- Existing theoretical works:

1. Y. B. Kim, A. Furusaki, X.-G. Wen, and P. A. Lee, Gauge- invariant response functions of fermions coupled to a gauge field, Phys. Rev. B **50**, 17917 (1994). $\sigma'(\Omega,0) \propto \frac{1}{\Omega^{2/3}}$

2. A. V. Chubukov and D. L. Maslov, Optical conductivity of a two-dimensional metal near a quantum critical point: The status of the extended Drude formula, Phys. Rev. B **96**, 205136 (2017). $\sigma'(\Omega,0) \propto \frac{1}{\Omega^{2/3}}$

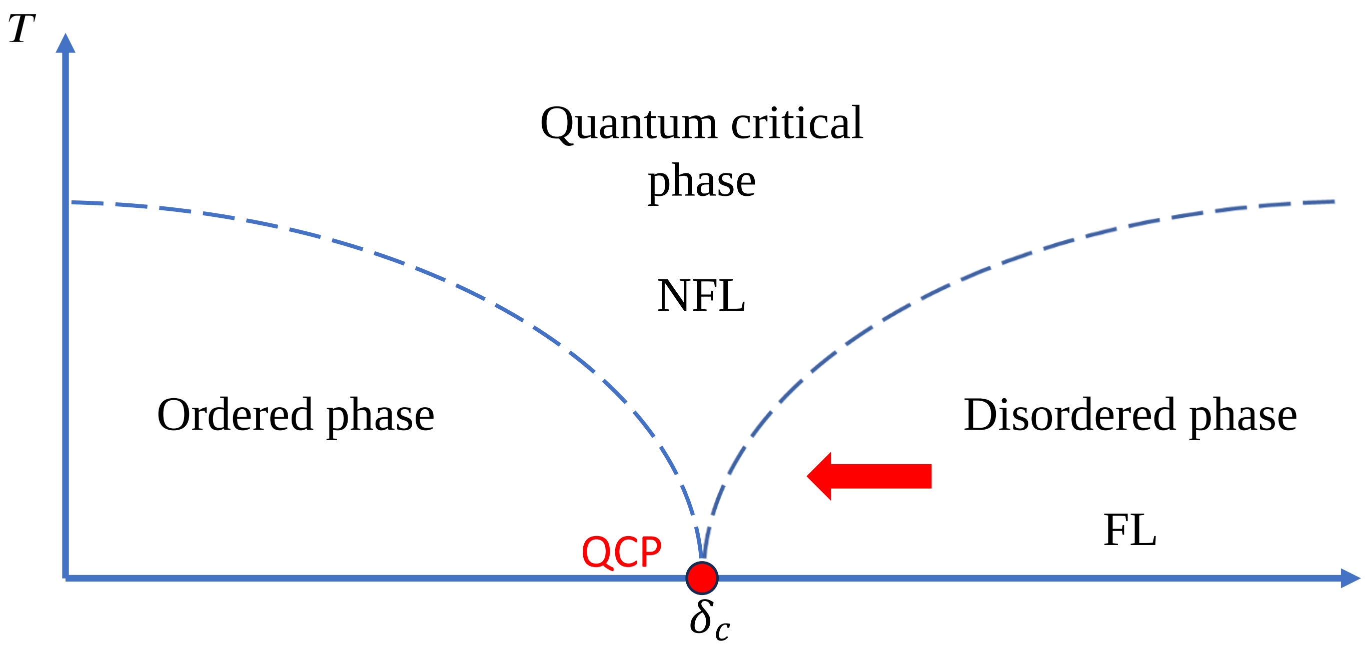
3. H. Guo, A. A. Patel, I. Esterlis, and S. Sachdev, Large-*N* theory of critical Fermi surfaces. II. Conductivity, Phys. Rev. B **106**, 115151 (2022). $\sigma'(\Omega,0) \propto \mathrm{const}$

4. Z. D. Shi, H. Goldman, D. V. Else, and T. Senthil, Gifts from anomalies: Exact results for Landau phase transitions in metals, SciPost Phys. **13**, 102 (2022).

5. Z. D. Shi, D. V. Else, H. Goldman, and T. Senthil, Loop current fluctuations and quantum critical transport, SciPost Phys. **14**, 113 (2023). $\sigma'(\Omega,0) \propto \frac{1}{\Omega^{2/3}}$

Part II: Optical conductivity near Ising-nematic QCP

- Quantum Critical Point (QCP)



δ: tuning parameter, e.g. pressure, doping, composition, magnetic field…

$z = 3$ QCP: $V(\boldsymbol{q}, \Omega_m) \sim \frac{1}{q^2 + \xi^{-2} + \gamma|\Omega_m|/q}$ (Landau damped critical boson, Hertz-Millis-Moriya theory)

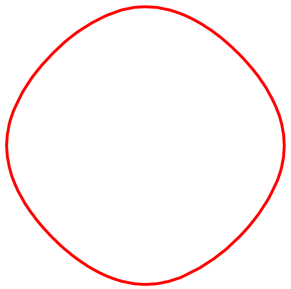
ξ: correlation length, $\xi \to \infty$ at QCP

Approach criticality from the FL side ($\xi < \infty$), crossover to quantum critical regime: $\xi^{-1} \to \max\{\omega^{\frac{1}{3}}, T^{\frac{1}{3}}\}$

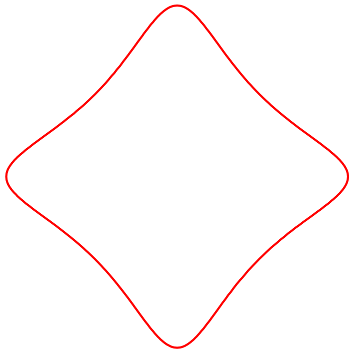
Songci Li, Prachi Sharma, Alex Levchenko, Dmitrii Maslov, Phys. Rev. B 108, 235125 (2023)
(see also Yasha Gindikin, Andrey Chubukov, Phys. Rev. B 109, 115156, (2024))

TABLE I. Optical conductivity of a 2D electron system near an Ising-nemtaic quantum critical point (QCP) for different types of Fermi surfaces. FL stands for the Fermi-liquid region.

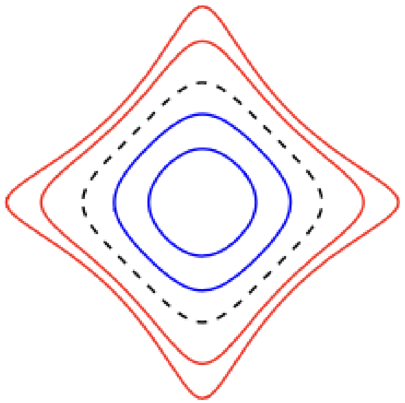
$\sigma'(\omega, T)$	Fermi surface			
	Isotropic with nonparabolic spectrum or convex		Concave	
	FL	QCP	FL	QCP
$\omega \gg T$	$\omega^2 \ln \lvert\omega\rvert$	$\lvert\omega\rvert^{2/3}$	const	$\lvert\omega\rvert^{-2/3}$
$\omega \ll T$	$T^4 \ln T/\omega^2$	$T^{8/3}/\omega^2$	T^2/ω^2	$T^{4/3}/\omega^2$



Convex FS



Concave FS
(with inflection points)



Crossover from
convex to concave FS

$\varepsilon(k) = -2t(cosk_x + cosk_y) +$
$4rtcosk_x\, cosk_y$
convex to concave transition: $\varepsilon =$
$\varepsilon_c = 8r(2r^2 - 1)$

1. Model Hamiltonian

$$H = H_0 + gH_{\text{int}},$$

$$H_0 = \sum_{\mathbf{k}s} \varepsilon_{\mathbf{k}} c^{\dagger}_{\mathbf{k},s} c_{\mathbf{k},s} \quad ,$$

$$H_{\text{int}} = \frac{1}{2} \sum_{\mathbf{q}} V(\mathbf{q}) d_{\mathbf{q}} d_{-\mathbf{q}}$$

$$= \frac{1}{2} \sum_{\mathbf{kpq}ss'} U(\mathbf{k}, \mathbf{p}, \mathbf{q}) c^{\dagger}_{\mathbf{k}_+,s} c^{\dagger}_{\mathbf{p}_-,s'} c_{\mathbf{p}_+,s'} \; c_{\mathbf{k}_-,s}$$

$$U(\mathbf{k}, \mathbf{p}, \mathbf{q}) = F(\mathbf{k}) F(\mathbf{p}) V(\mathbf{q}).$$

$F(\boldsymbol{k})$: Form factor projecting interaction into a given angular momentum channel

$$V(\mathbf{q}) = \frac{1}{q^2 + q_{\mathrm{B}}^2},$$

Orenstein-Zernike form, nearly critical FL

$$q_B \rightarrow \max\{\omega^{\frac{1}{3}}, T^{\frac{1}{3}}\}$$

2. Current density operator

$$\mathbf{j} = \mathbf{j}_0 + g\mathbf{j}_{\text{int}}$$

$$\mathbf{j}_0 = e \sum_{\mathbf{k}s} \boldsymbol{v}_{\mathbf{k}} c^{\dagger}_{\mathbf{k},s} c_{\mathbf{k},s}$$

$$\mathbf{j}_{\text{int}} = e \sum_{\mathbf{kpq}ss'} [(\boldsymbol{\nabla}_{\mathbf{k}} + \boldsymbol{\nabla}_{\mathbf{p}}) U(\mathbf{k}, \mathbf{p}, \mathbf{q})] c^{\dagger}_{\mathbf{k}_+,s} c^{\dagger}_{\mathbf{p}_-,s'} c_{\mathbf{p}_+,s'} \; c_{\mathbf{k}_-,s}$$

3. Kubo formula

$$\sigma'(\omega, T) \equiv \operatorname{Re} \sigma(\omega, T) = -\frac{1}{\omega} \operatorname{Im} \Pi_{\mathrm{j}}(\omega, T),$$

$$\Pi_{\mathrm{j}}(\omega, T) = -\frac{i}{2} \int_0^{\infty} dt e^{i\omega t} \langle [\mathbf{j}(t) \, ; \, \mathbf{j}(0)] \rangle$$

$$\equiv -\frac{i}{2} \langle [\mathbf{j}(t) \, ; \, \mathbf{j}(0)] \rangle_{\omega}$$

$$\boxed{\begin{aligned} \sigma'(\omega, T) &= \frac{1}{2\omega^3} \operatorname{Im} \langle [\mathbf{K}(t) \, ; \, \mathbf{K}(0)] \rangle_{\omega}, \\ \mathbf{K}(t) &= i\partial_t \mathbf{j} = [\mathbf{j}, H]. \end{aligned}}$$

Rosch, Howell, PRB 2005
Rosch, Ann. Phys. 2006

4. K operator

$$\mathbf{K}(t) = [\mathbf{j}_0 + g\mathbf{j}_{\mathrm{int}}, H_0 + gH_{\mathrm{int}}] = g\mathbf{K}_1(t) + g\mathbf{K}_2(t) + O(g^2),$$

$$\mathbf{K}_1(t) = [\mathbf{j}_0, H_{\mathrm{int}}],$$
$$\mathbf{K}_2(t) = [\mathbf{j}_{\mathrm{int}}, H_0].$$

$$\mathbf{K}_1 = -\frac{e}{2}\sum_{\mathbf{kpq}ss'} \Delta\boldsymbol{v}\, U(\mathbf{k},\mathbf{p},\mathbf{q}) c^\dagger_{\mathbf{k}_+,s} c^\dagger_{\mathbf{p}_-,s'} c_{\mathbf{p}_+,s'}\ \ c_{\mathbf{k}_-,s}$$

$$\mathbf{K}_2 = e\sum_{\mathbf{kpq}ss'} \Delta\varepsilon(\nabla_{\mathbf{k}} + \nabla_{\mathbf{p}}) U(\mathbf{k},\mathbf{p},\mathbf{q}) c^\dagger_{\mathbf{k}_+,s} c^\dagger_{\mathbf{p}_-,s'} c_{\mathbf{p}_+,s'}\ \ c_{\mathbf{k}_-,s}$$

$$\Delta\boldsymbol{v} = \boldsymbol{v}_{\mathbf{k}_+} + \boldsymbol{v}_{\mathbf{p}_-} - \boldsymbol{v}_{\mathbf{k}_-} - \boldsymbol{v}_{\mathbf{p}_+}$$

$$\Delta\varepsilon = \varepsilon_{\mathbf{k}_+} + \varepsilon_{\mathbf{p}_-} - \varepsilon_{\mathbf{k}_-} - \varepsilon_{\mathbf{p}_+}$$

5. Optical conductivity: $\sigma'(\omega,T) = \sigma'_1(\omega,T) + \sigma'_2(\omega,T) + \sigma'_{12}(\omega,T)$,

$$\sigma'_1(\omega,T) = \pi e^2 g^2 \frac{1-e^{-\omega/T}}{\omega^3}\int_{\mathbf{k},\mathbf{p},\mathbf{q}} U^2(\mathbf{k},\mathbf{p},\mathbf{q})(\Delta\boldsymbol{v})^2 M(\mathbf{k},\mathbf{p},\mathbf{q})\delta(\varepsilon_{\mathbf{k}_+} + \varepsilon_{\mathbf{p}_-} - \varepsilon_{\mathbf{k}_-} - \varepsilon_{\mathbf{p}_+} + \omega),$$

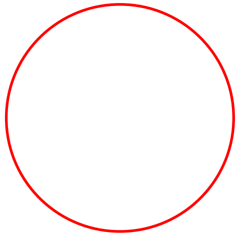
$$\sigma'_2(\omega,T) = 4\pi e^2 g^2 \frac{1-e^{-\omega/T}}{\omega^3}\int_{\mathbf{k},\mathbf{p},\mathbf{q}} [(\nabla_{\mathbf{k}} + \nabla_{\mathbf{p}})U(\mathbf{k},\mathbf{p},\mathbf{q})]^2(\Delta\varepsilon)^2 M(\mathbf{k},\mathbf{p},\mathbf{q})\delta(\varepsilon_{\mathbf{k}_+} + \varepsilon_{\mathbf{p}_-} - \varepsilon_{\mathbf{k}_-} - \varepsilon_{\mathbf{p}_+} + \omega),$$

$$\sigma'_{12}(\omega,T) = 4\pi e^2 g^2 \frac{1-e^{-\omega/T}}{\omega^3}\int_{\mathbf{k},\mathbf{p},\mathbf{q}} \Delta\varepsilon\Delta\boldsymbol{v}\cdot(\nabla_{\mathbf{k}} + \nabla_{\mathbf{p}})U^2(\mathbf{k},\mathbf{p},\mathbf{q}) M(\mathbf{k},\mathbf{p},\mathbf{q})\delta(\varepsilon_{\mathbf{k}_+} + \varepsilon_{\mathbf{p}_-} - \varepsilon_{\mathbf{k}_-} - \varepsilon_{\mathbf{p}_+} + \omega),$$

$$M(\mathbf{k},\mathbf{p},\mathbf{q}) = n_F(\varepsilon_{\mathbf{k}_+})n_F(\varepsilon_{\mathbf{p}_-})[1-n_F(\varepsilon_{\mathbf{k}_-})][1-n_F(\varepsilon_{\mathbf{p}_+})]$$

- Why does the geometry of FS matter? $\sigma'(\omega,T) \propto <\Delta v^2> \delta(\Omega+\varepsilon_k-\varepsilon_{k-q})\delta(\Omega-\omega-\varepsilon_p+\varepsilon_{p+q})$

$$\Delta \boldsymbol{v} = \boldsymbol{v}_k + \boldsymbol{v}_p - \boldsymbol{v}_{k-q} - \boldsymbol{v}_{p+q}$$



Isotropic FS
(nonparabolic)

$\boldsymbol{v}_{\mathbf{k}} = \epsilon'(k)\mathbf{k}/k$

- $\Delta\boldsymbol{v}$ vanishes identically for Galilean-invariant system (quadratic dispersion)
- Isotropic FS but nonparabolic dispersion: $\Delta\boldsymbol{v}$ vanishes if all momenta are placed onto the FS, need to expand around the FS

$$\Delta \boldsymbol{v} = \frac{w_{\mathrm{np}}}{k_{\mathrm{F}}}\left[(\varepsilon_{\mathbf{k}}-\varepsilon_{\mathbf{k-q}})\hat{\mathbf{k}} + (\varepsilon_{\mathbf{p}}-\varepsilon_{\mathbf{p+q}})\hat{\mathbf{p}} + (\varepsilon_{\mathbf{k-q}}-\varepsilon_{\mathbf{p+q}})\frac{\mathbf{q}}{k_{\mathrm{F}}}\right], \quad w_{\mathrm{np}} = 1 - \left.\frac{k_F\epsilon''(k)}{\epsilon'(k)}\right|_{k=k_F}$$

$$(\Delta \boldsymbol{v})^2 \sim w_{\mathrm{np}}^2 \max\{\omega^2, T^2\}/k_{\mathrm{F}}^2$$

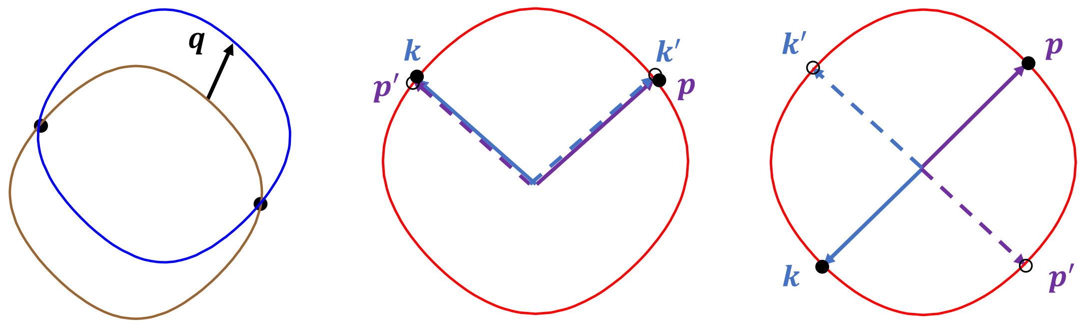
- Optical conductivity $\sigma'(\omega,T) \propto q_{\mathrm{B}}^{-4}\omega^2 \max\left\{\ln\frac{\omega_{\mathrm{FL}}}{|\omega|}, \frac{T^4}{\omega^4}\ln\frac{\omega_{\mathrm{FL}}}{T}\right\}.$

In terms of Drude formula:

$\sigma'(\omega,T) \propto \frac{1}{\omega^2\tau_J(\omega,T)}$, $\tau_J^{-1}(\omega,T) \propto 0 \times max\{\omega^2, T^2\} + \text{const} \times max\{\omega^4, T^4\}$

Extrapolating to QCP: $q_{\mathrm{B}} \to \max\{|\omega|^{1/3}, T^{1/3}\}$ $\Longrightarrow$ $\sigma'(\omega,0) \propto \omega^{2/3}$

- Why does the geometry of FS matter?



(a) Convex FS

(b) Swap channel, $\Delta \boldsymbol{v} = \mathbf{0}$

(c) Cooper channel, $\Delta \boldsymbol{v} = \mathbf{0}$

$$\sigma'(\omega, T) \propto < \Delta \boldsymbol{v}^2 > \delta(\varepsilon_{\boldsymbol{k}} - \varepsilon_{\boldsymbol{k}-\boldsymbol{q}})\delta(\varepsilon_{\boldsymbol{p}} - \varepsilon_{\boldsymbol{p}+\boldsymbol{q}})$$
$$\Delta \boldsymbol{v} = \boldsymbol{v}_{\boldsymbol{k}} + \boldsymbol{v}_{\boldsymbol{p}} - \boldsymbol{v}_{\boldsymbol{k}-\boldsymbol{q}} - \boldsymbol{v}_{\boldsymbol{p}+\boldsymbol{q}}$$

- $\varepsilon_{\boldsymbol{k}} = \varepsilon_{\boldsymbol{k}-\boldsymbol{q}}, \varepsilon_{\bar{\boldsymbol{p}}} = \varepsilon_{\bar{\boldsymbol{p}}-\boldsymbol{q}}$ ($\bar{\boldsymbol{p}} = -\boldsymbol{p}$, TRS)

intersection points of the original FS and the one shifted by $\boldsymbol{q}$, at most two intersection points (see (a)) for a convex FS

$$\varepsilon_{\mathbf{k}} = \varepsilon_{\mathbf{k}-\mathbf{q}} \Longrightarrow \{\mathbf{k}, -\mathbf{k}+\mathbf{q}\}$$
$$\varepsilon_{\bar{\mathbf{p}}} = \varepsilon_{\bar{\mathbf{p}}-\mathbf{q}} \Longrightarrow \{\bar{\mathbf{p}}, -\bar{\mathbf{p}}+\mathbf{q}\} = \{-\mathbf{p}, \mathbf{p}+\mathbf{q}\}$$

Two sets of solutions must coincide

- Only two possible choices:

1) swap scattering, $\boldsymbol{k} = \boldsymbol{p} + \boldsymbol{q}$ (see (b))

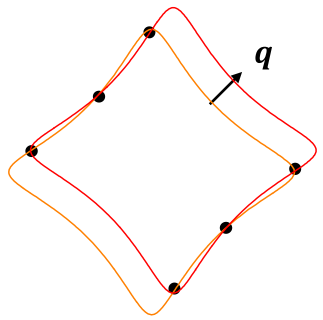
2) Cooper (head-on) scattering, $\boldsymbol{k} = -\boldsymbol{p}$ (see (c))

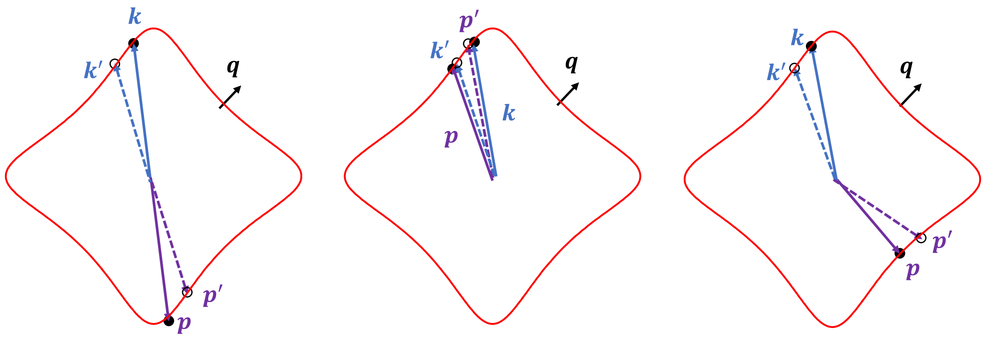
Both lead to $\Delta \boldsymbol{v} = 0$. Need to expand from the FS, $\Delta \boldsymbol{v}^2 \sim [\max(\omega, T)]^2$, similar to the isotropic case

$$\sigma'(\omega, T) \propto q_B^{-4}\omega^2 \max\left\{\ln\frac{\omega_{FL}}{|\omega|}, \frac{T^4}{\omega^4}\ln\frac{\omega_{FL}}{T}\right\}. \Longrightarrow \text{At QCP: } \sigma'(\omega, 0) \propto \omega^{2/3}$$

- Why does the geometry of FS matter?

$$\sigma'(\omega,T) \propto <\Delta \boldsymbol{v}^2> \delta(\varepsilon_{\boldsymbol{k}}-\varepsilon_{\boldsymbol{k-q}})\delta(\varepsilon_{\boldsymbol{p}}-\varepsilon_{\boldsymbol{p+q}})$$
$$\Delta \boldsymbol{v} = \boldsymbol{v}_{\boldsymbol{k}}+\boldsymbol{v}_{\boldsymbol{p}}-\boldsymbol{v}_{\boldsymbol{k-q}}-\boldsymbol{v}_{\boldsymbol{p+q}}$$





(a) Cooper channel, $\Delta \boldsymbol{v} = \boldsymbol{0}$

(b) Swap channel, $\Delta \boldsymbol{v} = \boldsymbol{0}$

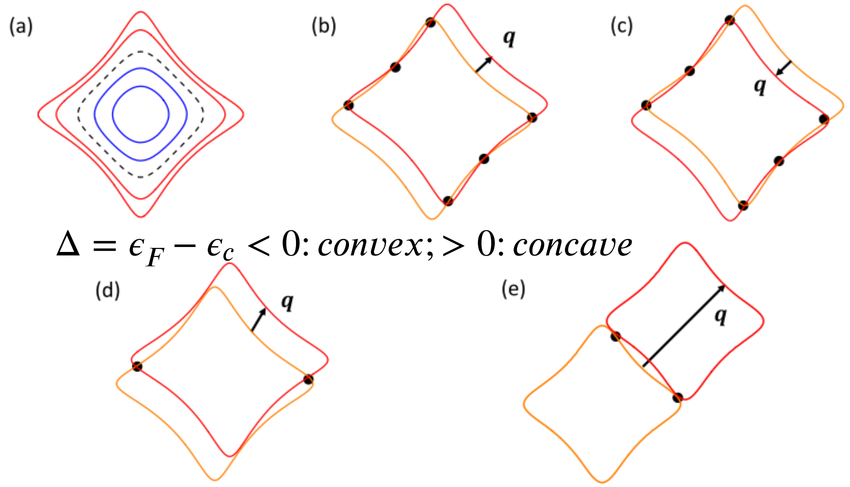
(c) Extra channel, $\Delta \boldsymbol{v} \neq \boldsymbol{0}$

Concave FS

- $N > 2$ intersection points for a concave FS
- Two of the channels are still head-on (a) and swap scattering (b), but rest of the scattering channels do relax current, $\Delta \boldsymbol{v} \neq 0$

$$\sigma'(\omega,T) \propto q_B^{-2}\frac{\omega^2+4\pi^2T^2}{\omega^2}$$

Extrapolating to QCP: $q_{\mathrm{B}} \to \max\{|\omega|^{1/3}, T^{1/3}\}$ $\Longrightarrow$ $\sigma'(\omega,0) \propto \omega^{-2/3}$



Convex to concave FS

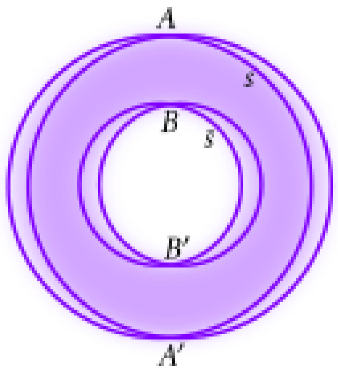
- Even for a concave FS, the kinematic constraint $\varepsilon_{\boldsymbol{k}} = \varepsilon_{\boldsymbol{k-q}}$ has more than 2 solutions only if $\boldsymbol{q}$ is near high symmetry axis and q being small

$$\Delta\phi_{\boldsymbol{q}} \propto \Delta^{3/2},\ q_{max} \propto \Delta^{1/2},\ (\Delta \boldsymbol{v})^2 \propto (q\Delta)^2$$

$$\sigma'(\omega,0) \propto \Theta(\Delta)\frac{\Delta^{7/2}}{\omega^{2/3}} + \omega^{2/3}$$

Yasha Gidishin, *Songci Li*, Alex Levchenko, Alex Kamenev, Andrey Chubukov, Dmitrii Maslov,
arXiv: 2406.10503, accepted in PRB

Two-valley system
(Multipli-connected FS)



Kinematic constraint $\varepsilon_{\boldsymbol{k}} = \varepsilon_{\boldsymbol{k}-\boldsymbol{q}}$ has $N > 2$ intersection points, similar to a concave FS
Expect in the FL regime:

$$\sigma'(\omega, T) \propto \frac{\omega^2 + 4\pi^2 T^2}{\omega^2}$$

$$\tau_J^{-1}(\omega, T) \propto \text{const} \times \; max\{\omega^2, T^2\}$$

We provide a diagrammatic derivation of optical conductivity based on Kubo formula

- One of the bands (band 1) is tuned near Ising-nematic QCP, while the other band (band 2) acts as a "momentum sink" for band 1 (two bands are isotropic but nonparabolic)

$$\mathcal{A} = \mathcal{A}_{\mathrm{f0}} + \mathcal{A}_{\mathrm{ff}},$$

$$\mathcal{A}_{\mathrm{f0}} = \sum_{j=1,2} \int d\tau \sum_{\mathbf{k}} \bar{c}_{\mathbf{k},j}(\tau) \left(\partial_\tau + \varepsilon_{\mathbf{k},j}\right) c_{\mathbf{k},j}(\tau)$$

$$\mathcal{A}_{\mathrm{ff}} = \frac{1}{2} \int d\tau \sum_{\mathbf{k},\mathbf{p},\mathbf{q}} \sum_{j j'=1,2} F(\mathbf{k}) F(\mathbf{p}) v_{jj'}$$
$$\times \bar{c}_{\mathbf{k}+\frac{\mathbf{q}}{2},j}(\tau) \bar{c}_{\mathbf{p}-\frac{\mathbf{q}}{2},j'}(\tau) c_{\mathbf{p}+\frac{\mathbf{q}}{2},j'}(\tau) c_{\mathbf{k}-\frac{\mathbf{q}}{2},j}(\tau)$$

$$\hat{v} = \begin{pmatrix} v_{11} & v_{12} \\ v_{12} & v_{22} \end{pmatrix},$$

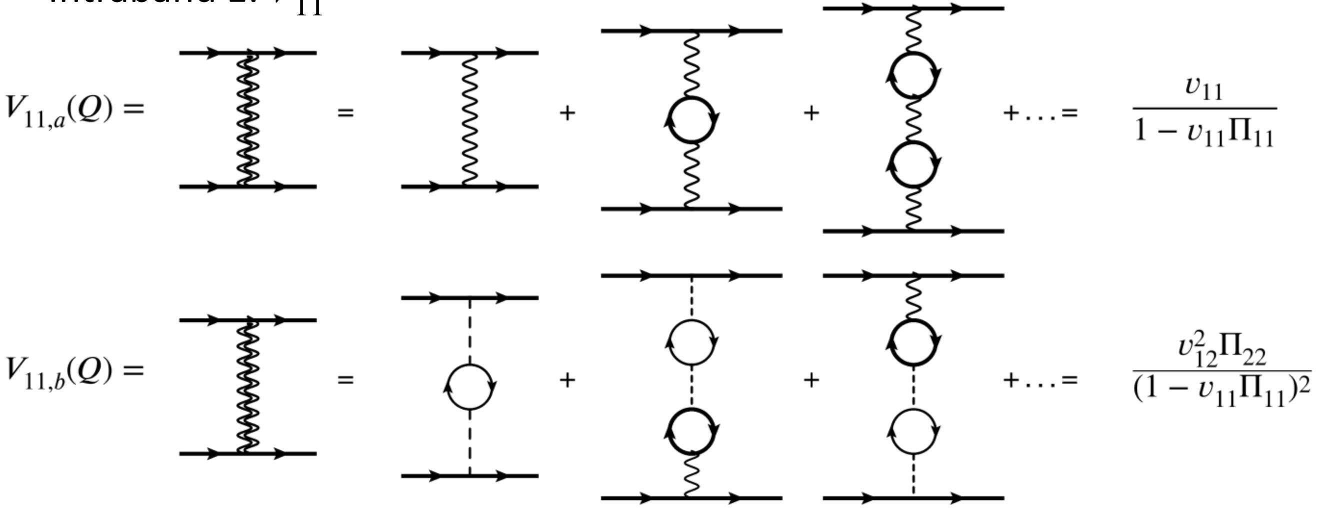
Bare Hubbard interaction

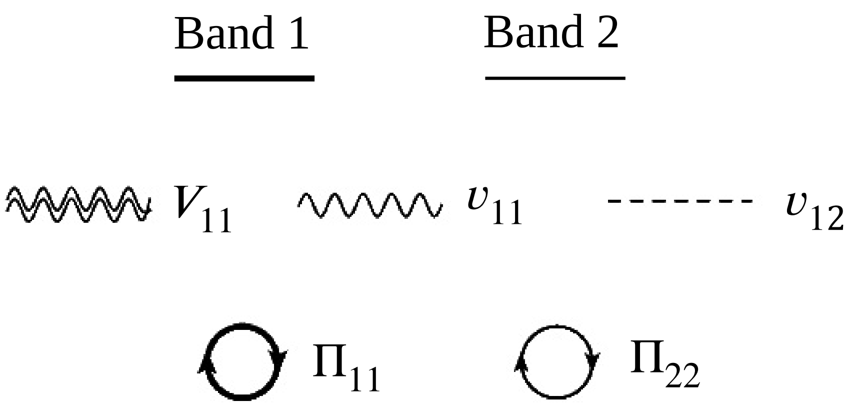
- RPA for dressed interaction $\hat{V}$

$$\hat{V}(\mathbf{q}, \Omega_m) = \hat{v} + \hat{v}\hat{\Pi}(\mathbf{q}, \Omega_m)\hat{V}(\mathbf{q}, \Omega_m),$$
$$\hat{\Pi}(\mathbf{q}, \Omega_m) = \mathrm{diag}\left(\Pi_{11}(\mathbf{q}, \Omega_m), \Pi_{22}(\mathbf{q}, \Omega_m)\right)$$

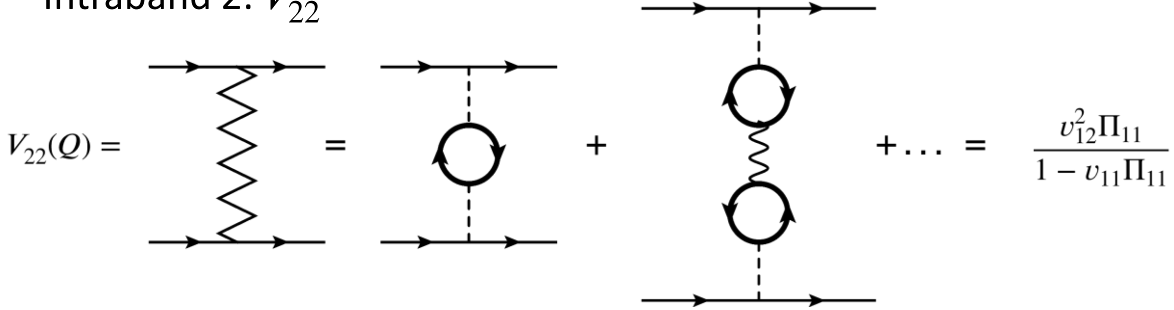
Assuming band 2 is noninteracting, $v_{22} = 0$;
Expand to $O(v_{12}^2)$

(a) Intraband 1: V_{11}



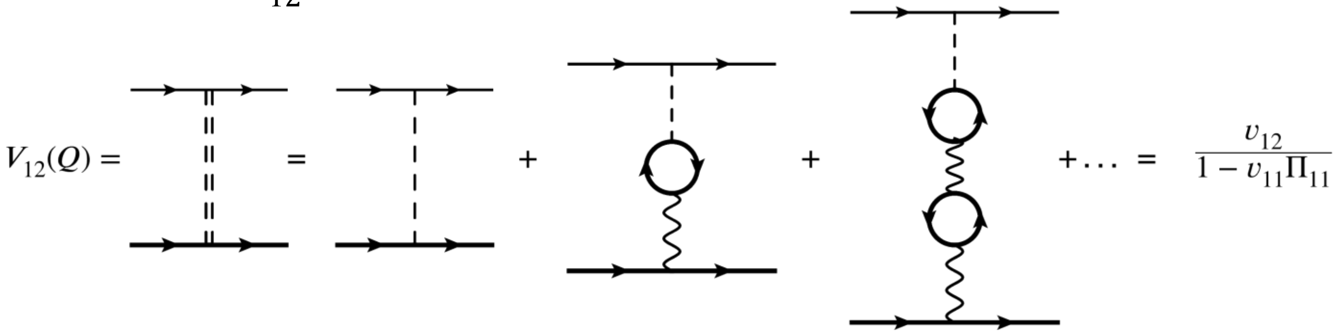


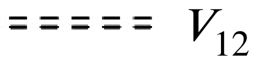
(b) Intraband 2: V_{22}



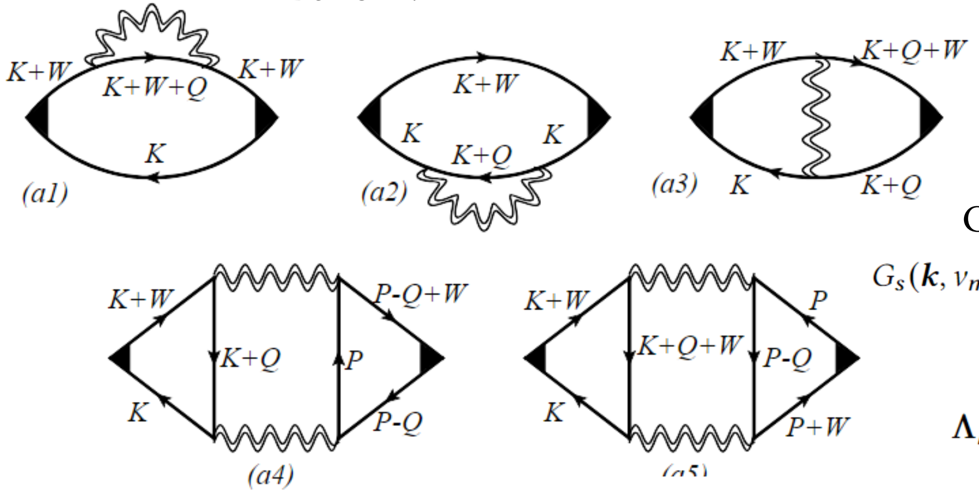
- Though the bare intra-band interaction in band 2 was neglected, the coupling to band 1 gives rise to an effective interaction within band 2

(c) Interband: V_{12}

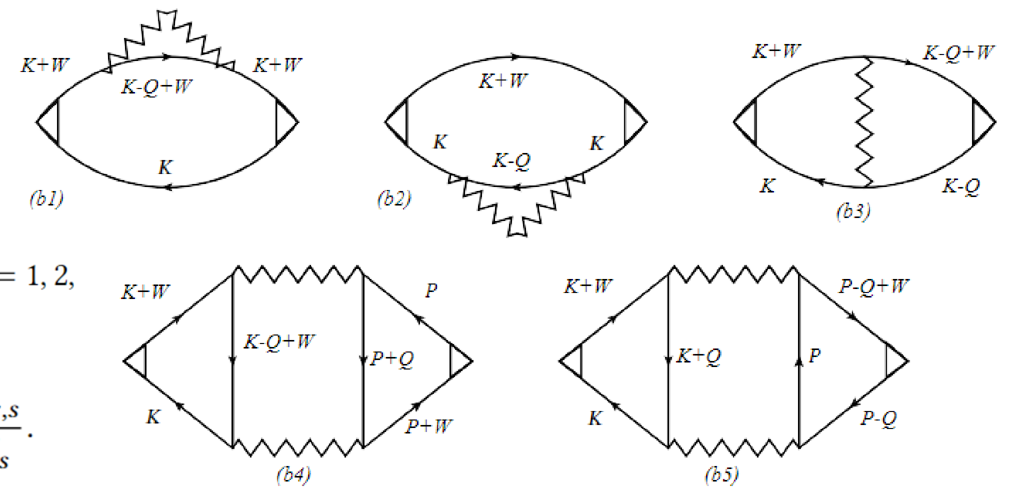




a) Band 1:



b) Band 2:



Green's function:

$$G_s(\boldsymbol{k},\nu_m) = \frac{1}{\frac{i\nu_m}{Z_s} - \varepsilon_{\boldsymbol{k},s}},\quad s = 1, 2,$$

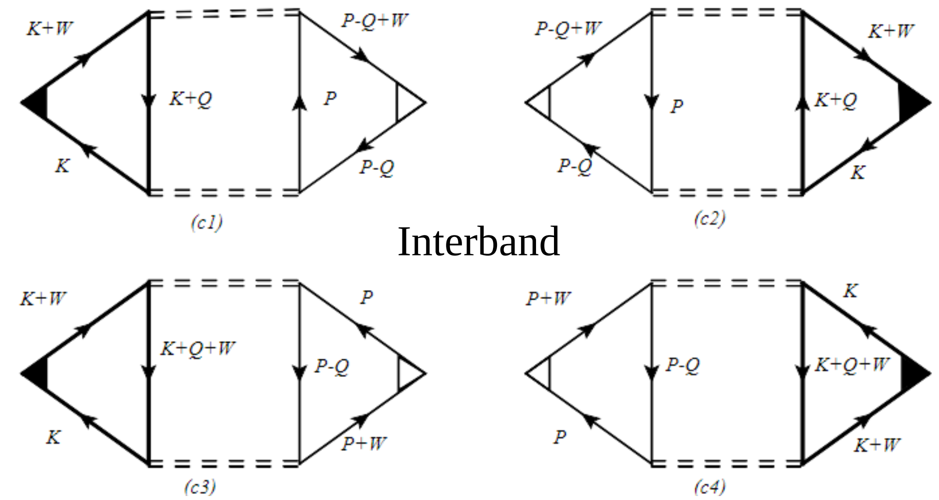
Current Vertex:

$$\boldsymbol{\Lambda}_{\boldsymbol{k},s} = \boldsymbol{v}_{\boldsymbol{k},s}\Gamma_s = \frac{\boldsymbol{v}_{\boldsymbol{k},s}}{Z_s}.$$

$$a) + b):\quad \mathrm{Re}\,\sigma_{\mathrm{intra}}(\omega) = A_D e^2 \lambda_{12} \frac{q^{*4}}{v_{\mathrm{F},1} v_{\mathrm{F},2} q_{\mathrm{B}}^{4-D}} \left(\frac{1}{\bar{m}_1^2} + \frac{1}{\bar{m}_2^2}\right)$$

dimensionless interband coupling $\lambda_{12} = N_{F1} N_{F2} v_{12}^2$

c)



Interband

$$c):\quad \mathrm{Re}\,\sigma_{\mathrm{inter}}(\omega) = -2 A_D e^2 \lambda_{12} \frac{q^{*4}}{\bar{m}_1 \bar{m}_2 v_{\mathrm{F},1} v_{\mathrm{F},2} q_{\mathrm{B}}^{4-D}}.$$

$$\boxed{\begin{aligned}\mathrm{Re}\,\sigma(\omega) &= \mathrm{Re}\,\sigma_{\mathrm{intra}}(\omega) + \mathrm{Re}\,\sigma_{\mathrm{inter}}(\omega)\\ &= A_D e^2 \left(\frac{1}{\bar{m}_1} - \frac{1}{\bar{m}_2}\right)^2 \lambda_{12} \frac{q^{*4}}{v_{\mathrm{F},1} v_{\mathrm{F},2} q_{\mathrm{B}}^{4-D}}.\end{aligned}}$$

Crossover to QCP: $q_B \to \omega^{1/3}$ ➡

$$\boxed{\mathrm{Re}\,\sigma(\omega) \propto \left(\frac{1}{\bar{m}_1} - \frac{1}{\bar{m}_2}\right)^2 \lambda_{12} |\omega|^{-(4-D)/3}.}$$

2D:
$\sigma'(\omega) \propto \omega^{-2/3}$

Part III: Summary

- Optical conductivity $\sigma'(\omega, T)$of a NFL: a 2D clean metal near Ising-nematic QCP ($z = 3, q = 0$), ω- and T-dependence sensitive to the geometry of Fermi surface

$\sigma'(\omega, T)$	Fermi surface			
	Isotropic with nonparabolic spectrum or convex		Concave	
	FL	QCP	FL	QCP
$\omega \gg T$	$\omega^2 \ln \lvert\omega\rvert$	$\lvert\omega\rvert^{2/3}$	const	$\lvert\omega\rvert^{-2/3}$
$\omega \ll T$	$T^4 \ln T/\omega^2$	$T^{8/3}/\omega^2$	T^2/ω^2	$T^{4/3}/\omega^2$

Convex FS: $\sigma'(\omega, 0) \propto \omega^{2/3}$ Concave FS: $\sigma'(\omega, 0) \propto \omega^{-2/3}$

- Optical conductivity $\sigma'(\omega, T)$of a two-valley system near Ising-nematic QCP

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Thank you for your attention!